UNIVERSITY OF COLOMBO, SRI LANKA

UCSC UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

BIT

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2010/2011 – 2nd Year Examination – Semester 3

IT3304: Mathematics for Computing-II

PART 2 - Structured Question Paper

25th February 2011

(ONE HOUR)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **07 pages**.
- **Answer all questions.**
- **Question 2 (**40% marks**) and other questions (**30% marks each**).**
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. X) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

1)

(a) Let A = (a_{ij}) be a square matrix of order n and C_{ij} be the cofactor of a_{ij} .

(i) Write an expression to find |A| by expanding along the row i.

(ii) Write an expression to find |A| by expanding along the column j.

(b) Let $A=\frac{1}{3}\begin{pmatrix} 11 & -2 & 8 & 5 \\ -4 & 2 & -6 & 2 \\ 8 & 1 & 6 & 9 \\ -7 & 12 & 3 & 6 \end{pmatrix}, B=\frac{1}{3}\begin{pmatrix} -8 & 3 & 9 & -2 \\ 3 & -5 & 2 & -3 \\ -7 & 10 & -6 & -8 \\ 6 & 1 & 4 & -7 \end{pmatrix}, C=\frac{1}{3}\begin{pmatrix} 9 & 11 & 7 & 2 \\ 0 & 1 & 8 & 12 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

Find

(i) $|\mathbf{A}+\mathbf{B}|$

(ii) $|\mathbf{C}|$

(c) Let $A=\frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$.

Find

(i) A^{-1}

(ii) $|A^{-1}|$

(30 marks)

ANSWER IN THIS BOX

(a)

(i) $|A| = \sum_{j=1}^{n} a_{ij}C_{ij}$

(ii) $|A| = \sum_{i=1}^{n} a_{ij}C_{ij}$

(b) (i) $|A+B| = \left|\frac{1}{3}\begin{pmatrix} 3 & 1 & 17 & 3 \\ -1 & -3 & -4 & -1 \\ 1 & 11 & 0 & 1 \\ -1 & 13 & 7 & -1 \end{pmatrix}\right| = 0$ as 1st and 4th columns are equal

(ii) $|C|=1/3$

(c) (i) $C = \frac{1}{9}\begin{pmatrix} -6 & 6 & -3 \\ -3 & -6 & -6 \\ -6 & -3 & 6 \end{pmatrix}$

$$\textbf{adj } \mathbf{A} = \mathbf{C^T} = \frac{1}{9}\begin{pmatrix} -6 & -3 & -6 \\ 6 & -6 & -3 \\ -3 & -6 & 6 \end{pmatrix}$$

$$= -\frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

$|A| = -1$

$$\mathbf{A^{-1} = (adj\ A)/|A| =} \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

(ii) $|A A^{-1}| = |A||A^{-1}| = 1$
$|A^{-1}| = -1$

2) (a) Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$

(b) Find the area bounded by the curves $y_1 = x^2$ and $y_2 = 1 + 2x - x^2$.

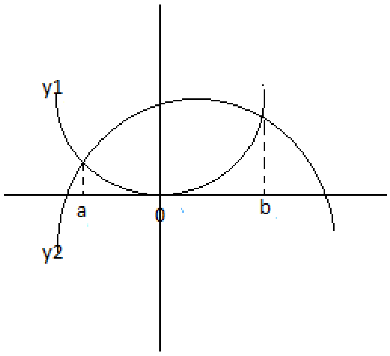
(40 marks)

ANSWER IN THIS BOX

(a)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \lim_{k\to\infty} \sum_{n=1}^{k} \frac{1}{n(n+1)} = \lim_{k\to\infty} \sum_{n=1}^{k} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \lim_{k\to\infty}\left(1 - \frac{1}{k+1}\right) = 1.$$

(b)



$y_1 = y_2$ gives $x = a = \frac{1-\sqrt{3}}{2}$ and $x = b = \frac{1+\sqrt{3}}{2}$.

Hence the required area is

$$\int_a^b (y_2 - y_1)dx = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{1+\sqrt{3}}{2}} (1 + 2x - 2x^2)dx = x + x^2 - 2x^3/3\Big|_{\frac{1-\sqrt{3}}{2}}^{\frac{1+\sqrt{3}}{2}}$$

$$= \left(\frac{1+\sqrt{3}}{2}\right) + \left(\frac{1+\sqrt{3}}{2}\right)^2 - \frac{2}{3}\left(\frac{1+\sqrt{3}}{2}\right)^3 - \left(\frac{1-\sqrt{3}}{2}\right) - \left(\frac{1-\sqrt{3}}{2}\right)^2 + \frac{2}{3}\left(\frac{1-\sqrt{3}}{2}\right)^3$$

$= \sqrt{3}$

3) In a game, a player tosses three different coins and counts the number of heads obtained. The probability distribution of the number of heads obtained is given below.

X	0	1	2	3
P[X= x]	c	c^2	$c - 2c^2$	$c^2 + 2c$

(a) Determine the value of *c*.

(b) Calculate the probability of getting at most one head.

(c) If that player gets more than two heads he will win the game. Calculate the probability of winning the game.

(d) Calculate the expected number of heads that the player can get.

(e) Calculate the standard deviation of the number of heads obtained.

(30 marks)

ANSWER IN THIS BOX

(a)

$$\begin{aligned} c + c^2 + c - 2c^2 + c^2 + 2c &= 1 \\ 4c &= 1 \\ c &= \frac{1}{4} = 0.25 \end{aligned}$$

(b)

P[at most one head] = $P[X = 0] + P[X = 1] = 0.25 + 0.0625 = 0.3125$

(c)

P{winning the game} = P[more than two heads]

$= P[X = 2] + P[X = 3] = 0.125 + 0.5625 = 0.6875$

Or (by using part (b)

P{winning the game} = P[more than two heads]

= 1-P[at most one head] = 1- 0.3125 = 0.6875

(d)

$$\begin{aligned} E(X) &= 0*(0.25) + 1*(0.0625) + 2*(0.125) + 3*(0.5625) \\ &= 0 + 0.0625 + 0.250 + 1.6875 \\ &= 2 \end{aligned}$$

(e)

$$\begin{aligned} E(X^2) &= 0^2 *(0.25) + 1^2 *(0.0625) + 2^2 *(0.125) + 3^2 *(0.5625) \\ &= 0 + 0.0625 + 0.5 + 5.0625 \\ &= 5.625 \end{aligned}$$

$$V(X) = E(X^2) - [E(X)]^2 = 5.625 - 2^2 = 1.625$$

Standard Deviation = $\sqrt{V(X)} = \sqrt{1.625} = 1.2747$

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