UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UCSC

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2011/2012 – 2nd Year Examination – Semester 3

IT3304: Mathematics for Computing-II

PART 2 - Structured Question Paper

24th February 2012

(ONE HOUR)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is 1 (One) hour.
- The medium of instruction and questions is English.
- This paper has 3 questions and 09 pages.
- Answer all questions.
- Question 1 (40% marks) and other questions (30% marks each).
- Write your answers in English using the space provided in this question paper.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. X) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

1) Suppose A and B are two square matrices each of order n. When is B said to be the inverse of A?

(5marks)

Let $A = \begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & 3 & -3 \\ 3 & 3 & -1 & -1 \end{pmatrix}$.

Find

(i) $A(A)^T$

(10marks)

(ii) $(A)^T A$

(10marks)

(iii) Does A^{-1} exist? If your answer is yes, find it.

(15marks)

ANSWER IN THIS BOX

B is said to be the inverse of A when AB = BA = I where I is the identity matrix of order n.

(i)

$$\mathbf{AA^T} = \begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & 3 & -3 \\ 3 & 3 & -1 & -1 \end{pmatrix}\begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ 1 & 3 & 3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 20 \end{pmatrix}$$

(ii)

$$\mathbf{A^TA} = \begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ 1 & 3 & 3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix}\begin{pmatrix} 3 & -3 & 1 & -1 \\ 1 & 1 & 3 & 3 \\ -1 & 1 & 3 & -3 \\ 3 & 3 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 20 \end{pmatrix}$$

(iii) $A^{-1} = \dfrac{1}{20}\begin{pmatrix} 3 & 1 & -1 & 3 \\ -3 & 1 & 1 & 3 \\ 1 & 3 & 3 & -1 \\ -1 & 3 & -3 & -1 \end{pmatrix}$

2)

(a) Let S_n denote the n^{th} partial sum of the series $\sum_{n=1}^{\infty} 3n\left(\frac{1}{2}\right)^{n-1}$.

By considering $S_n - \frac{1}{2} S_n$ prove that $S_n = 12 - (12+6n)\left(\frac{1}{2}\right)^n$.

Hence prove that $\sum_{n=1}^{\infty} 3n\left(\frac{1}{2}\right)^{n-1} = 12$.

(20 marks)

(b) Evaluate $\displaystyle\int_1^{\sqrt{3}} \frac{1}{x^2(x^2+1)}\,dx$.

(10 marks)

ANSWER IN THIS BOX

(a) $S_n = 3 + 3(2)\left(\frac{1}{2}\right) + 3(3)\left(\frac{1}{2}\right)^2 + 3(4)\left(\frac{1}{2}\right)^3 + \ldots\ldots + 3(n)\left(\frac{1}{2}\right)^{n-1}$

$\therefore\ \frac{1}{2}S_n = 3\left(\frac{1}{2}\right) + 3(2)\left(\frac{1}{2}\right)^2 + 3(3)\left(\frac{1}{2}\right)^3 + 3(4)\left(\frac{1}{2}\right)^4 + \ldots\ldots + 3(n)\left(\frac{1}{2}\right)^n$

Therefore

$S_n - \frac{1}{2}S_n = 3 + 3\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \ldots\ldots + 3\left(\frac{1}{2}\right)^{n-1} - 3n\left(\frac{1}{2}\right)^n$

$\therefore\ \frac{1}{2}S_n = 3\left[\frac{1-(1/2)^n}{1-1/2}\right] - 3n\left(\frac{1}{2}\right)^n$

$\therefore\ S_n = 12 - (12+6n)\left(\frac{1}{2}\right)^n.$

$\therefore\ \displaystyle\sum_{n=1}^{\infty} 3n\left(\frac{1}{2}\right)^{n-1} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[12 - (12+6n)\left(\frac{1}{2}\right)^n\right] = 12.$

Continued…

3) The number of claims for missing baggage in a day at a particular airport in a small city is a discrete random variable with the following probability distribution function.

X	0	1	2	3	4	5	6	7	8	9	≥10
Probability	0.05	$2a$	0.15	0.1	A	0.15	0.05	b	0.05	0.05	0

(a) It is given that the probability of X being greater than or equal to 5 is 0.4. Calculate the values of a and b.

(b) Calculate the probability of getting at most one claim.

(c) Calculate the probability of getting at least 6 claims.

(d) Calculate the probability of getting less than 4 claims.

(e) Calculate the probability of the number of claims being between 3 and 6 inclusive.

(f) Calculate the expected number of claims.

(30 marks)

ANSWER IN THIS BOX

(a) $P(X \geq 5)$ $= P(5) + P(6) + P(7) + P(8) + P(9)$

$= 0.15 + 0.05 + b + 0.05 + 0.05$

$= 0.3 + b = 0.4$

ð $b = 0.1$

$P(X \leq 4)$ $= P(0) + P(1) + P(2) + P(3) + P(4)$

$= 0.05 + 2a + 0.15 + 0.1 + a$

$= 3a + 0.3 = 0.6$

ð $3a = 0.3$

ð $a = 0.1$

(b) $P(X \leq 1)$ $= P(0) + P(1)$

$= 0.05 + 0.2$

$= 0.25$

(c) $P(X \geq 6)$ $= P(6) + P(7) + P(8) + P(9)$

$= 0.05 + 0.1 + 0.05 + 0.05$

$= 0.25$

(d) $P(X < 4)$ $= P(0) + P(1) + P(2) + P(3)$

$= 0.05 + 0.2 + 0.15 + 0.1$

$= 0.5$

(e) $P(3 \leq X \leq 6) = P(3) + P(4) + P(5) + P(6)$

$= 0.1 + 0.1 + 0.15 + 0.05$

$= 0.4$

(f) $E(X) = 0P(0) + 1P(1) + 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6) + 7P(7) + 8P(8) + 9P(9)$

$= (0*0.05) + (1*0.2) + (2*0.15) + (3*0.1) + (4*0.1) + (5*0.15) + (6*0.05) + (7*0.1) + (8*0.05) + (9*0.05)$

$= 0 + 0.2 + 0.3 + 0.3 + 0.4 + 0.75 + 0.3 + 0.7 + 0.4 + 0.45$

$= 3.8$

Continued…

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