UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UCSC

BIT

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2009/2010 – 2nd Year Examination – Semester 3

IT3303: Mathematics for Computing-II

PART 2 - Structured Question Paper

19th March 2010

(ONE HOUR)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **8 pages**.
- **Answer all questions.**
- **Question 2 (**40% marks**) and other questions (**30% marks**).**
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. X) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

1) Consider the following system of three linear equations.

$$\begin{aligned} x + 2y - 3z &= -1 \\ 3x - y + 2z &= 8 \\ 5x + 3y - 4z &= 6 \end{aligned}$$

(i) Transform this system of linear equations into matrix form and identify the coefficient matrix.

(ii) Let the coefficient matrix be denoted by *A*. Is *A* invertible? Justify your answer.

(iii) Write down the **three elementary row operations** used in matrix algebra.

(iv) Apply elementary row operations to solve the given system of linear equations.

(v) Is the given system of linear equations consistent? Justify your answer.

(30 Marks)

ANSWER IN THIS BOX

(i) $$\begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ 5 & 3 & -4 \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \\ 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ 5 & 3 & -4 \end{pmatrix}$$

(ii) Not invertible. |A| = 0.

(iii) 1 Interchanging two rows of a matrix

2 Multiplying a row by a non-zero constant

3 Adding a non-zero multiple of one row to another row

Continued…

(iv) Multiplying the first row by –3 and adding it to the second row we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 5 & 3 & -4 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 6 \end{pmatrix}$$

Multiplying the first row by –5 and adding it to the third row we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 0 & -7 & 11 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 11 \end{pmatrix}$$

Multiplying row 2 by –1 and adding to row 3 we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 0 \end{pmatrix}$$

Multiplying row 2 by –1/7

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -11/7 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ -11/7 \\ 0 \end{pmatrix}$$

Multiplying row 2 by –2 and adding to row 1, we obtain

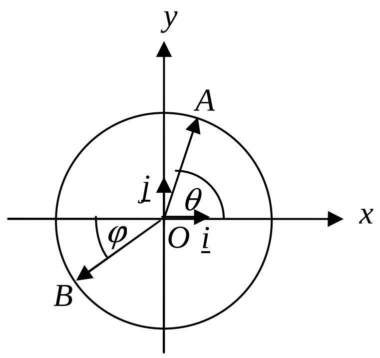
$$\begin{pmatrix} 1 & 0 & 1/7 \\ 0 & 1 & -11/7 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} 15/7 \\ -11/7 \\ 0 \end{pmatrix}$$

This system has infinitely many solutions of the form

z = k, y= $\frac{11}{7}(\boldsymbol{k}-1)$ x= $\frac{1}{7}(15-\boldsymbol{k})$

(v) Consistent. As shown in (iv), it has infinitely many solutions.

2) Consider the follow figure of a unit circle with centre at the origin and $0 < \theta,\ \varphi < \dfrac{\pi}{2}$.



(i) Write down the position vector of A and of B in terms of the unit vectors $\underline{i}$ and $\underline{j}$.

(ii) Find the dot product $\overrightarrow{OA}\,.\,\overrightarrow{OB}$ and the cross product $\overrightarrow{OA} \times \overrightarrow{OB}$.

(iii) Using (ii), prove that

(a) $\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$ and

(b) $\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$

(40 Marks)

ANSWER IN THIS BOX

(i) $\overrightarrow{OA} = \cos\ \theta\,\underline{i} + \sin\ \theta\,\underline{j}$

$\overrightarrow{OB} = -\cos\ \varphi\,\underline{i} - \sin\ \varphi\,\underline{j}$

(ii) $\overrightarrow{OA}.\overrightarrow{OB} = (\cos\ \theta\,\underline{i} + \sin\ \theta\,\underline{j}).(-\cos\ \varphi\,\underline{i} - \sin\ \varphi\,\underline{j})$

$= -\cos\theta\cos\varphi - \sin\theta\sin\varphi$ -------------- (1)

Continued…

$$\vec{OA} \times \vec{OB} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos\theta & \sin\theta & 0 \\ -\cos\varphi & -\sin\varphi & 0 \end{vmatrix}$$

$= (-\ \cos\ \theta \sin\ \varphi\ +\ \sin\ \theta \cos\ \varphi)\ \underline{k}$ ---------------- (2)

(iii) $\vec{OA} . \vec{OB} = |\vec{OA}|\,|\vec{OB}| \cos\gamma$ where γ is the angle between $\vec{OA}$ and $\vec{OB}$.

$|\vec{OA}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$, $\quad |\vec{OB}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$.

Therefore,

$\vec{OA} . \vec{OB} = (1)(1)\cos(\pi + \varphi - \theta) = -\cos(\varphi - \theta) = -\cos(\theta - \varphi)$ ---------
(3)

By (1) and (3), $-\cos(\theta - \varphi) = -\cos\theta\cos\varphi - \sin\theta\sin\varphi$

Therefore, $\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$

$\vec{OA} \times \vec{OB} = |\vec{OA}|\,|\vec{OB}| \sin\gamma\ \underline{k} = (1)\,(1)\sin(\pi + \varphi - \theta)\ \underline{k}$

$= -\sin(\theta - \varphi)\ \underline{k}$ ------------- (4)

By (2) and (4), $-\sin(\theta - \varphi) = (-\ \cos\ \theta \sin\ \varphi\ +\ \sin\ \theta\ \cos\ \varphi)$

Therefore, $\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$

Continued…

3) Let X be the continuous random variable which is defined as '***the length of a random access memory (RAM) card***', in centimeters (cm), produced by a particular manufacturer. The probability density function (pdf) of X is given as follows.

$$f(x) = \begin{cases} kx & 0 \le x \le 6 \\ 0 & otherwise \end{cases}$$

(i) Calculate the value of k.
(ii) Calculate the probability that the length of a RAM card is not more than 5 cm.
(iii) Calculate the probability that the length of a RAM card is at least 5 cm.
(iv) Evaluate the expected value of X.
(v) Evaluate the standard deviation of X.

(30 Marks)

ANSWER IN THIS BOX

(i)

$$\text{since} \int_{-\infty}^{+\infty} f(x)dx = 1$$

$$\int_0^6 kxdx = 1$$

$$k\int_0^6 xdx = 1$$

$$k\left[\frac{x^2}{2}\right]_0^6 = 1$$

$$k\left[\frac{6^2}{2} - 0\right] = 1$$

$$k\frac{36}{2} = 1$$

$$18k = 1$$

$$\underline{\underline{k = \frac{1}{18}}}$$

Continued…

(ii)

$$P[X \le 5] = \int_0^5 \frac{1}{18} x\,dx = \frac{1}{18}\int_0^5 x\,dx = \frac{1}{18}\left[\frac{x^2}{2}\right]_0^5$$

$$= \frac{1}{18}\left[\frac{5^2}{2} - 0\right] = \frac{25}{36}$$

$$= 0.6944$$

$$\therefore \quad P[X \le 5] = 0.6944$$

(iii)

$$P[X \ge 5] = 1 - P[X < 5] = 1 - \frac{25}{36} = \frac{11}{36}$$

$$= 0.3056$$

$$\therefore P[X \ge 5] = 0.3056$$

(iv)

$$E[X] = \int_0^6 x \frac{1}{18} x\,dx = \frac{1}{18}\int_0^6 x^2\,dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_0^6 = \frac{1}{18}\left[\frac{6^3}{3} - 0\right]$$

$$= \frac{6\times 6\times 6}{18\times 3} = \frac{216}{54}$$

$$= 4$$

$$\therefore \quad E[X] = 4cm$$

(v)

$$E[X^2] = \int_0^6 x^2 \frac{1}{18} x\,dx = \frac{1}{18}\int_0^6 x^3\,dx = \frac{1}{18}\left[\frac{x^4}{4}\right]_0^6 = \frac{1}{18}\left[\frac{6^4}{4} - 0\right]$$

$$= \frac{6\times 6\times 6\times 6}{18\times 4} = \frac{1296}{72}$$

$$= 18$$

$$\therefore \quad E[X^2] = 18$$

$$V[X] = E[X^2] - [E[X]]^2$$

$$= 18 - 4^2$$

$$= 18 - 16$$

$$= 2$$

Therefore standard deviation = $\sqrt{2} = 1.414$ cm

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