UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UCSC

BIT

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2012/2013 – 2nd Year Examination – Semester 3

IT3304: Mathematics for Computing-II

PART 2 - Structured Question Paper

01st March 2013

(ONE HOUR)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **11 pages**.
- **Answer all questions.**
- **Question 1 carries** 40% marks **and the other questions carry** 30% marks each.
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. X) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

Index No: ………………..

1) State the possible types of solutions that a consistent system of linear equations can have.

(5 marks)

Consider the following system of linear equations in matrix form?

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ 5 & 3 & -4 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \\ 6 \end{pmatrix}$$

Let A = $\begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ 5 & 3 & -4 \end{pmatrix}$.

(i) Does A^{-1} exist? Justify your answer.

(15 marks)

(ii) Show that the given system of linear equations has infinitely many solutions.

(20 marks)

ANSWER IN THIS BOX

The types solutions are Unique solutions and Infinitely many solutions

(i) No. |A|=0

(ii)

Multiplying the first row by –3 and adding it to the second row we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 5 & 3 & -4 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 6 \end{pmatrix}$$

Multiplying the first row by –5 and adding it to the third row we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 0 & -7 & 11 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 11 \end{pmatrix}$$

Multiplying row 2 by –1 and adding to row 3 we obtain

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -7 & 11 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \\ 0 \end{pmatrix}$$

Multiplying row 2 by –1/7

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -\tfrac{11}{7} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -1 \\ -\tfrac{11}{7} \\ 0 \end{pmatrix}$$

Multiplying row 2 by –2 and adding to row 1, we obtain

$$\begin{pmatrix} 1 & 0 & \tfrac{1}{7} \\ 0 & 1 & -\tfrac{11}{7} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} \tfrac{15}{7} \\ -\tfrac{11}{7} \\ 0 \end{pmatrix}$$

This system has infinitely many solutions of the form

z = k, y= $\frac{11}{7}(\boldsymbol{k}-1)$ x= $\frac{1}{7}(15-\boldsymbol{k})$, k $\in$ R.

Index No: ………………..

Index No: ………………..

2) (a) Let $f(x) = mx^{\alpha} - x, \quad x > 0$ where m is a positive number and $0 < \alpha < 1$.

(i) Find the stationary point of $f(x)$.

(ii) Show that the maximum value of $f(x)$ is

$$\left(m^{\frac{1}{1-\alpha}}\right)\left(\alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{1}{\alpha}-1\right)$$

(15 marks)

(b) $\underline{x}$ and $\underline{y}$ are vectors where $|\underline{x}| = 5$. Find $|\underline{x} + \underline{y}|$ if

(i) $\underline{x}$ is perpendicular to $\underline{y}$ and $|\underline{y}| = 5$.

(ii) $\underline{y}$ is perpendicular to $\underline{x} + \underline{y}$ and $|\underline{y}| = 3$.

(15 marks)

<u>ANSWER IN THIS BOX</u>

1 (a) Let $f(x) = \mathrm{m}x^{\alpha} - x, \quad x > 0$ where m is a positive number and $0 < \alpha < 1$.

(i) $f'(x) = \mathrm{m}\alpha x^{\alpha-1} - 1 = 0 \rightarrow x = \left(\frac{1}{\mathrm{m}\alpha}\right)^{\frac{1}{\alpha-1}}$.

$\therefore$ Stationary point $x^* = \left(\frac{1}{\mathrm{m}\alpha}\right)^{\frac{1}{\alpha-1}}$.

(ii) $f''(x^*) = \mathrm{m}\alpha(\alpha-1)(x^*)^{\alpha-2} < 0$ since $0 < \alpha < 1$.

Therefore f is a maximum at x^* and the maximum value of f is

$$f(x^*) = \mathrm{m}(x^*)^{\alpha} - x^* = \mathrm{m}\left(\frac{1}{\mathrm{m}\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\mathrm{m}\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$= \left(\frac{1}{\mathrm{m}\alpha}\right)^{\frac{1}{\alpha-1}}\left[m\frac{1}{m\alpha} - 1\right]$$

$$= \left(m^{\frac{1}{1-\alpha}}\right)\left(\alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{1}{\alpha}-1\right)$$

Index No: ………………..

(b)

(i) If $\underline{x}$ is perpendicular to $\underline{y}$ then we have

$$|\underline{x}+\underline{y}|^2 = |\underline{x}|^2 + |\underline{y}|^2 = 5^2 + 5^2.$$

Therefore $|\underline{x}+\underline{y}| = 5\sqrt{2}.$

(ii) If $\underline{y}$ is perpendicular to $\underline{x}+\underline{y}$ then we have

$$|\underline{x}|^2 = |\underline{x}+\underline{y}|^2 + |\underline{y}|^2 .$$

Hence $|\underline{x}+\underline{y}| = \sqrt{|\underline{x}|^2 - |\underline{y}|^2} = \sqrt{5^2-3^2} = 4..$

Continued…

Index No: ………………..

Index No: ………………..

3) The time taken to download a certain type of virus guard follows a normal distribution with mean *72 seconds* and variance *36 seconds*. (The Standard Normal Distribution table is attached herewith).

(a) Calculate the probability that the time taken to download this type of virus guard is more than *75 seconds*.

(b) Calculate the probability that the time taken to download this type of virus guard is between *72 seconds* and *75 seconds*.

(c) What is the maximum time it would take to download 95% of this type of virus guard?

(30 marks)

ANSWER IN THIS BOX

(a) Let X: Time taken to download this type of virus guard

Then $X \sim N(\mu=72, \sigma^2=36)$

$$
\begin{aligned}
P[X > 75] &= P\left[\frac{X-\mu}{\sigma} > \frac{75-72}{6}\right] \\
&= P\left[Z > \frac{3}{6}\right] \\
&= P[Z > 0.5] \\
&= 1 - P[Z < 0.5] \\
&= 1 - 0.6915 \\
&= 0.3085
\end{aligned}
$$

(b)

$$
\begin{aligned}
P[72 < X < 75] \quad &= P\left[\frac{72-72}{6} < \frac{X-\mu}{\sigma} < \frac{75-72}{6}\right] \\
&= P\left[0 < Z < \frac{3}{6}\right] \\
&= P[0 < Z < 0.5] \\
&= P[Z < 0.5] - P[Z < 0] \\
&= 0.6915 - 0.5000 \\
&= 0.1915
\end{aligned}
$$

(c)

$P[X < x] \quad = 0.95$

From the table

$P[Z < z] \quad = 0.95$

$z = 1.96$

that is;

$\frac{x-\mu}{\sigma} = 1.96$

$\frac{x-72}{6} = 1.96$

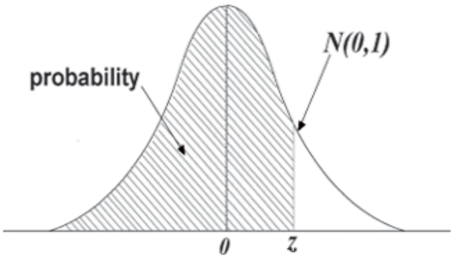
$x = (1.96 \times 6) + 72$

$= 11.76 + 72$

$= 83.76$

*****

The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/ \sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	***P***	***Z***	***P***	***Z***	***P***
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997