UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2013/2014 – 2nd Year Examination – Semester 3

IT3304: Mathematics for Computing-II

PART 2 - Structured Question Paper

28th February 2014

(ONE HOUR)

To be completed by the candidate

BIT Examination Index No: .....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **10 pages**.
- **Answer all questions**.
- **Question 1 carries** 40% marks **and the other questions carry** 30% marks each.
- **Write your answers** in English using the space provided **in this question paper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. [X]) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

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1)

(a) Consider the following system of linear equations:

$$\begin{pmatrix} A_{11} & A_{12} & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & A_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ A_{m1} & A_{m2} & . & . & A_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_m \end{pmatrix}$$

(i) When is this system said to be homogeneous?

(05 marks)

(ii) Is a homogeneous system of linear equations consistent? Justify your answer.

(10 marks)

(b) Consider the following system of three linear equations.

$$\begin{aligned} 5x - 3y - z &= 16 \\ 3x - 2y + 2z &= 5 \\ 2x + y + 3z &= 5 \end{aligned}$$

(i) Transform this system of linear equations into matrix form.

(05 marks)

(ii) Using elementary row operations solve the given system of linear equations.

(15 marks)

(iii) Does the system have no solution, a unique solution or infinitely many solutions?

(05 marks)

ANSWER IN THIS BOX

(1) (a)

(i) $y_1 = y_2 = y_3 = \ldots\ldots\ldots\ldots = y_m = 0$

(ii) Yes. It has at least one solution which is $x_1 = x_2 = x_3 = \ldots\ldots\ldots\ldots = x_n = 0$

(b)

(i)

$$\begin{bmatrix}5 & -3 & -1\\ 3 & -2 & 2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}16\\ 5\\ 5\end{bmatrix}$$

(ii)

Multiplying the 1st row by 1/5

$$\begin{bmatrix}1 & -3/5 & -1/5\\ 3 & -2 & 2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}16/5\\ 5\\ 5\end{bmatrix}$$

Multiplying 1st row by -3 and adding to 2nd row/ multiplying1st row by -2 and adding to 3rd row

$$\begin{bmatrix}1 & -3/5 & -1/5\\ 0 & -1/5 & 13/5\\ 0 & 11/5 & 17/5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}16/5\\ -23/5\\ -7/5\end{bmatrix}$$

Multiplying 2nd row by –5

$$\begin{bmatrix}1 & -3/5 & -1/5\\ 0 & 1 & -13\\ 0 & 11/5 & 17/5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}16/5\\ 23\\ -7/5\end{bmatrix}$$

Multiplying 2nd row by 3/5 and adding to 1st row/ multiplying 2nd row by -11/5 and adding to 3rd row

$$\begin{bmatrix}1 & 0 & -8\\ 0 & 1 & -13\\ 0 & 0 & 32\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}17\\ 23\\ -52\end{bmatrix}$$

Multiplying 3rd row by 1/32

$$\begin{bmatrix}1 & 0 & -8\\ 0 & 1 & -13\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}17\\ 23\\ -13/8\end{bmatrix}$$

Multiplying 3rd row by 13 and adding to 2nd row/ multiplying 3rd row by 8 and adding to 1st row

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 15/8\\ -13/8\end{bmatrix}$$

x=32/8, y=15/8, z= -13/8

(iii) The system has a unique solution

.

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2) (a) A circle of radius 10 cm is divided into 12 sectors such that the angles of the sectors are in an arithmetic progression. The angle at the centre of the largest sector is 8 times that of the smallest sector. Find the perimeter of the largest sector.

(b) The first two terms of a geometric progression are 1 and $\frac{1}{3}\tan^2\theta$ respectively, where $0 < \theta < \frac{\pi}{2}$.

i. Determine the set of values of θ for which the progression is convergent.

ii. Find the infinite sum of the progression when $\theta = \frac{\pi}{6}$

(c) Find the area bounded by the curve $y = \sqrt{x+1}$ and the line $y = x+1$.

(3*10 marks)

ANSWER IN THIS BOX

(a) a, a+d,……..,a+11d is the progression.

But a+11d=8a

Therefore 7a-11d=0 ← (1)

Also S_{12}=360.

Hence $\frac{12}{2}[2a+11d] = 360$ which gives

2a+11d=60 ← (2)

By (1) and (2) we have a = 20/3 and d=140/33. The perimeter of the largest sector is

P = 2r+rθ

= 20+10×(a+11d)/2π

$=20+\frac{10}{2\pi}\left[\frac{20}{3}+\frac{140}{3}\right] = 20+\frac{800}{3\pi}$.

(b) a=1 and $r=\frac{1}{3}\tan^2\theta$

(i)For convergence, lrl<1 $\rightarrow \frac{1}{3}\tan^2\theta < 1$

$\rightarrow \tan^2\theta < 3$

$\rightarrow -\sqrt{3} < \tan\theta < \sqrt{3}$

$\therefore 0 < \theta < \frac{\pi}{3}$

(ii) $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}\tan^2\left(\frac{\pi}{6}\right)} = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}.$

(**c**) Area $= \int_{-1}^{0}\left[\sqrt{x+1}-(x+1)\right]dx$

$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{x^2}{2} - x\right]_{-1}^{0}$

$= \frac{2}{3} - \left(0 - \frac{1}{2} + 1\right)$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$

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3) The weight of a snack packet is normally distributed with mean of *100grams* and a standard deviation of *5grams*.

(a) Calculate the probability that the weight of a snack packet is more than *110grams*.

(b) Calculate the probability that the weight of a snack packet is between *90grams* and 105*grams*.

(c) 5% of the snack packets has weight each less than *m* grams. Find *m*.

(3*10 marks)

ANSWER IN THIS BOX

(a) $P[X > 110] = P\left[\frac{X-\mu}{\sigma} > \frac{110-100}{5}\right] = P[Z > 2] = 1 - P[Z < 2] = 1 - 0.9772 = 0.0228$

(b) $P[90 < X < 105] = P\left[\frac{90-100}{5} < \frac{X-\mu}{\sigma} < \frac{105-100}{5}\right]$

$= P[-2 < Z < 1] = P[Z < 1] - P[Z < -2] = 0.8413 - 0.0228 = 0.8185$

(c)

$From\ table\ \ z = -1.64$

$Therefore\ \frac{x-\mu}{\sigma} = -1.64$

$\frac{x-100}{5} = -1.64$

$x = 100 - 1.64 * 5$

$x = 100 - 8.2$

$x = 91.8$

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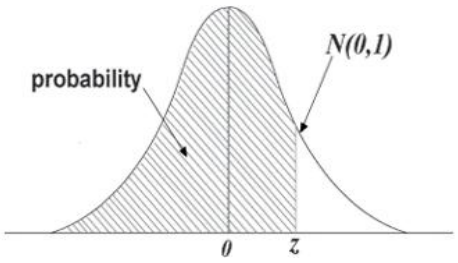
The Standard Normal Distribution Table



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/ \sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997