UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2014/2015 – 2nd Year Examination – Semester 3

IT3305: Mathematics for Computing-II

PART 2 - Structured Question Paper

15th March 2015

(ONE HOUR)

To be completed by the candidate
BIT Examination Index No: ....

Important Instructions:

- The duration of the paper is **1 (One) hour**.
- The medium of instruction and questions is English.
- This paper has **3 questions** and **11 pages**.
- **Answer all questions**.
- **Question 1 carries** 40% marks **and the other questions carry** 30% marks each.
- **Write your answers** in English using the space provided **in this questionpaper**.
- Do not tear off any part of this answer book.
- Under no circumstances may this book, used or unused, be removed from the Examination Hall by a candidate.
- Note that questions appear on both sides of the paper.
  If a page is not printed, please inform the supervisor immediately.

Questions Answered

Indicate by a cross (×), (e.g. ☒) the numbers of the questions answered.

To be completed by the candidate by marking a cross (×).	1	2	3	
To be completed by the examiners:				

1)

(a) Consider three matrices, $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & 0\\ -2 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$.

Verify that $[A(BC)]^T = (C^T B^T)A^T$ by evaluating both the matrix expressions separately

(10 marks)

ANSWER IN THIS BOX

(1) (a)

$$BC=\begin{bmatrix}3 & 0\\ -2 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}6 & -3\\ -3 & 3\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}\begin{bmatrix}6 & -3\\ -3 & 3\end{bmatrix}=\begin{bmatrix}0 & 3\\ -15 & 12\end{bmatrix}$$

$$[A(BC)]^T=\begin{bmatrix}0 & -15\\ 3 & 12\end{bmatrix}\text{======================(1)}$$

$$C^T B^T=\begin{bmatrix}2 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}3 & -2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}6 & -3\\ -3 & 3\end{bmatrix}$$

$$(C^T B^T)A^T=\begin{bmatrix}6 & -3\\ -3 & 3\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}=\begin{bmatrix}0 & -15\\ 3 & 12\end{bmatrix}\text{========================(2)}$$

By (1) and (2), $[A(BC)]^T = (C^T B^T)A^T$

(b)

(i). Define the inverse of a matrix.

(05 marks)

ANSWER IN THIS BOX

(b) (i)

Let A be an n x n square matrix. If there exists an n x n matrix B such that $AB = BA = I_{n \times n}$ where $I_{n \times n}$ is the n x n identity matrix, B is said to be the inverse of A.

(ii). Let $A = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$.

(I) Show that the product of the two matrices A and A^T, is commutative.

(05 marks)

ANSWER IN THIS BOX

(b) (ii)(I)

$$AA^T = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}\frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

$$AA^T = \frac{1}{9}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$$

$$A^T A = \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}_3 \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix} = I_3$$

Therefore, $AA^T = A^TA$

(II) Find A^{-1}.

(5 marks)

ANSWER IN THIS BOX

(b) (ii)(II)

$$A^T A = \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix} \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix} = I_3$$

$$AA^T = A^T A = I_3$$

$$\therefore A^{-1} = A^T = \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

(c) Consider the following system of linear equations.

$$\begin{aligned} 2x + y - z &= 3 \\ x + 3y + 2z &= -1 \\ 2x + 2y + z &= 2 \end{aligned}$$

Solve the above equations using matrix operations.

(15 marks)

ANSWER IN THIS BOX

(c)

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ 2 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

R1=R1*1/2

$$\begin{pmatrix} 1 & 1/2 & -1/2 \\ 1 & 3 & 2 \\ 2 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3/2 \\ -1 \\ 2 \end{pmatrix}$$

R2=R2+R1*(-1); R3=R3+R1*(-2)

$$\begin{pmatrix} 1 & 1/2 & -3/2 \\ 0 & 5/2 & 5/2 \\ 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 3/2 \\ -5/2 \\ -1 \end{pmatrix}$$

R2=R2*(2/5)

$$\begin{pmatrix} 1 & 1/2 & -3/2 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 3/2 \\ -1 \\ -1 \end{pmatrix}$$

R1=R1+R2*(-1/2); R3=R3+R2*(-1)

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

R2=R2+ R3*(-1); R1=R1+R3*(2)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

Therefore, z=0, y=-1, x=2

2) (a) To which value does the sequence $x_n = \frac{2n-1}{2n+1}$ converge, as n tends to infinity? Justify your answer.

(10 marks)

ANSWER IN THIS BOX

(a) $x_n = \frac{2n-1}{2n+1}$

$= \frac{2-\frac{1}{n}}{2+\frac{1}{n}} \to \frac{2-0}{2+0}$= 1as n tends to infinity,

(b) If $\sin x \approx x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$, find an approximate expansion for $\cos x$.

(10 marks)

ANSWER IN THIS BOX

(b) On differentiation, $\cos x \cong 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$.

(c) Find the area in the first quadrant bounded by the x-axis, y-axis and the curve $y = 4 - x^2$.

(10 marks)

<u>ANSWER IN THIS BOX</u>

(c) $\int_0^2 (4 - x^2)\, dx$

$= \left[4x - \frac{x^3}{3}\right]_0^2$

$= 8 - \frac{8}{3}$

$= \frac{16}{3}$

3)

The number of emails received per day by a first year undergraduate student in a certain university is a discrete random variable with the following probability distribution function.

X	0	1	2	3	4	5	6	7	More than 7
Probability	a	2a	0.15	0.10	a	0.15	0.10	b	0

(a) It is given that the probability of X is less than or equals 2, is 0.3. Calculate the values of *a* and *b*.

(05 marks)

<u>ANSWER IN THIS BOX</u>

(a) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.3$

$a + 2a + 0.15 = 0.3$

$3a + 0.15 = 0.3$

$3a = 0.15$

$a = 0.05$

Continue…

Total probability =1.0

$$4a + b + 0.5 = 1$$
$$0.2 + b + 0.5 = 1.0$$
$$b = 0.75$$

For a particular day, calculate the following for questions (b) to (f).

(b) Calculate the probability of getting at least one email.

(05 marks)

ANSWER IN THIS BOX

$P(at\ least\ one) = P(X \geq 1) = 1 - P(X = 0)$

$= 1 - a$

$= 1 - 0.05 = 0.95$

(c) Calculate the probability of getting at most 3 emails.

(05 marks)

ANSWER IN THIS BOX

$$\begin{aligned} P(at\ most\ 3) &= P(X \leq 3) \\ &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\ &= a + 2a + 0.15 + 0.10 \\ &= 0.15 + 0.15 + 0.10 \\ &= 0.40 \end{aligned}$$

(d) Calculate the probability of getting more than 4 emails.

(05 marks)

ANSWER IN THIS BOX

$$\begin{aligned} P(more\ than\ 4) &= P(X > 4) \\ &= 1 - P(X \leq 3) \\ &= 1 - (0.40) \\ &= 0.60 \end{aligned}$$

(e) Calculate the probability of getting between 2 and 5 exclusive emails.

(05 marks)

ANSWER IN THIS BOX

$$P(2 < X < 5) = P(X = 3) + P(X = 4)$$
$$= 0.10 + a$$
$$= 0.10 + 0.05$$
$$= 0.15$$

(f) Calculatethe expected number of emails.

(05 marks)

ANSWER IN THIS BOX

$$E(X) = \sum X.P(X)$$

$$= (0)(0.05) + (1)(0.10) + (2)(0.15) + (3)(0.10) + (4)(0.05) + (5)(0.15) + (6)(0.10) + (7)(0.75) + 0$$

$$= 7.5$$

*****